On the instanton-induced portion of the nucleon strangeness

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Abstract. We calculate the instanton contribution to the proton strangeness in an MIT bag enriched by the presence of a dilute instanton liquid. The evaluation is based on the expression of the nucleon matrix elements of bilinear strange-quark operators in terms of a model valence nucleon state and the interactions producing quark–antiquark fluctuations on top of that valence state. Our method combines use of the the evolution operator containing a strangeness source and the Feynman–Hellmann theorem. It enables one to evaluate the strangeness in different Lorentz channels in essentially the same way. Only the scalar channel is found to be affected by the interaction induced by the random instanton liquid.

1 Introduction

Despite the accumulated evidence for the nucleon strangeness, there has been as yet no balanced understanding of its various appearances. By a particular nucleon strangeness we mean the value of the nucleon matrix element $\langle N|\mathcal{O}_s(\Gamma)|N\rangle$, where the bilinear $\mathcal{O}_s(\Gamma) = \bar{s}\Gamma s$ might represent scalar, pseudoscalar, vector, axial vector or tensor strange-current densities ($\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$). Thus, any interaction $\mathcal{L}_I$ that induces $s\bar{s}$ pairs in the nucleon state $|N\rangle$ potentially leads to various types of nucleon strangeness. The imaginable interactions $\mathcal{L}_I$, which are related to QCD-vacuum fluctuations, might prefer some of the strangeness channels. In particular, there is a conjecture [1] that a nontrivial QCD-vacuum structure selects the pseudoscalar and scalar channels, which experience the axial and trace anomaly, respectively. In the present paper we focus on QCD-vacuum fluctuations as given by the instanton-liquid model [2–4], i.e., we take $\mathcal{L}_I \rightarrow \mathcal{L}_{\text{inst}}$. Such an interaction generates an s-quark loop (schematically shown in Fig. 1) to which an external probe can couple. It is important that this interaction can be treated perturbatively; this enables us to compare its relative contributions to different strangeness channels. The relatively complicated interaction $\mathcal{L}_I = \mathcal{L}_{\text{inst}}$ (given by (22)–(24) below) is conveniently split into three pieces,

$$\mathcal{L}_I = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 , \tag{1}$$

where the parts illustrated in Fig. 1 refer to the one-, two- and three-body operators. These operators change the known valence (model) state $|N_0\rangle$ to the state $|N\rangle$
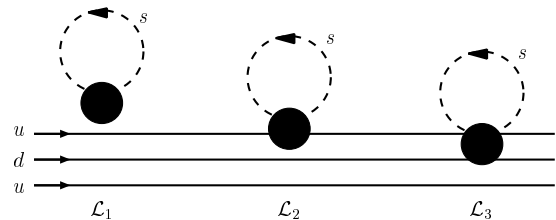
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Fig. 1. Instanton-induced local strangeness represented by the effective one-, two- and three-body operators. Non-strange quarks are denoted by solid lines, and strange ones by dashed lines

containing the $s\bar{s}$ pairs. Then we provide an expression (19) suitable for computing the strange matrix element of the full nucleon state, $\langle N| : \bar{s}\Gamma s : |N\rangle$.

The current evidence for the strangeness content of the proton comes from the external probe at both low- and high-momentum transfers. The analysis of the term $\sigma_{\pi N}$ in low-energy πN scattering reveals comparable light- and strange-quark nucleon matrix elements [1] (N means the proton throughout this paper):

$$\langle N|\bar{u}u|N\rangle \simeq 4.8 ,$$
$$\langle N|\bar{d}d|N\rangle \simeq 4.1 ,$$
$$\langle N|\bar{s}s|N\rangle \simeq 2.8 , \tag{2}$$

i.e., the unexpectedly large *scalar strangeness*. *A posteriori*, it is found to be in accordance with QCD-vacuum characteristics [1], as represented, for example, by the (naive) bag-model relation [5]

$$\langle N|\bar{s}s|N\rangle = -\langle 0|\bar{s}s|0\rangle V, \tag{3}$$

or the QCD sum-rule result [6] :

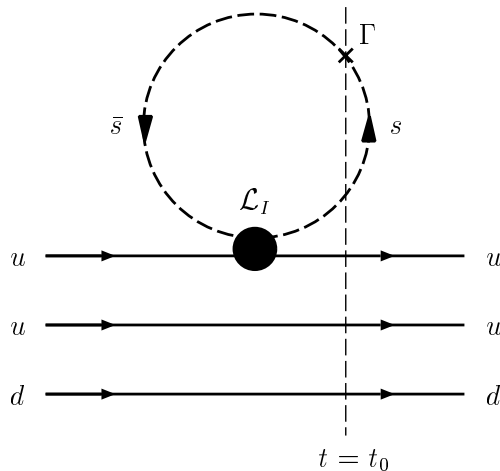$$\langle N|\bar{s}s|N\rangle \simeq 2.4 . \tag{4}$$

Fig. 2. Nonvanishing nucleon strangeness due to a response of the valence nucleon state to a strangeness source at Γ (denoted by $\times$), i.e., to a probe coupled to strange quarks through Γ. More precisely, this graph is that part of the nucleon response which arises only through one interaction $\mathcal{L}_I$

The other piece of evidence for the strangeness content of the proton comes from the polarized lepton–nucleon scattering at relatively high-momentum transfer, higher than the scales pertinent for our considerations. The analysis [7] of new data supports the original EMC findings [8,9], revealing a nonvanishing fraction $\Delta s = -0.11 \pm 0.06$ of the proton spin S_μ carried by the s quark. This is significant in comparison with $\Delta u = 0.82 \pm 0.06$ and $\Delta d = -0.44 \pm 0.06$. Δs is related to the *axial strangeness* of the proton, defined as $\langle N | \bar{s} \gamma_\mu \gamma_5 s | N \rangle = \Delta s \, S_\mu$.

The *vector strangeness*, described by the Dirac and the Pauli form factors as

$$\langle N | \bar{s} \gamma_\mu s | N \rangle = \bar{u}_N(p') \left[F_1^s(q^2) \gamma_\mu + F_2^s(q^2) \frac{\mathrm{i}\sigma_{\mu\nu} q^\nu}{2M_N} \right] u_N(p),$$
(5)

can be related to the analogous flavour singlet (0) and the hypercharge (8) form factors for $\langle N | V_\mu^{(0,8)} | N \rangle$ through

$$\bar{s} \gamma_\mu s = V_\mu^{(0)} - 2 V_\mu^{(8)},$$
$$V_\mu^{(0)} = \frac{1}{3} \bar{q} \gamma_\mu q, \quad V_\mu^{(8)} = \frac{1}{\sqrt{3}} \bar{q} \gamma_\mu \frac{\lambda^8}{2} q.$$
(6)

Although $F_1^s(0) = 0$ is the net nucleon strangeness, its momentum dependence determines the strangeness radius

$$r_s^2 = 6 \frac{\mathrm{d}}{\mathrm{d}q^2} F_1^s(q^2) \bigg|_{q^2=0},$$
(7)

while the strange magnetic moment is given by

$$\mu_s = F_2^s(0).$$
(8)

Note the relation $F_2^s = F_2^{(0)} - (2/\sqrt{3}) F_2^{(8)}$, where the last term is constrained by $(2/\sqrt{3}) F_2^{(8)} = \kappa_p + \kappa_n = -0.12$.

Knowledge of the $F_2^{(0)}$ flavour singlet term would also enable one to determine the baryomagnetic moment

$$\mu_p^{(0)} = F_1^{(0)}(0) + F_2^{(0)}(0) = (1 + F_2^{(0)}) \, \mu_N$$
(9)

There are many varying model-dependent calculations [10–22] of the quantities listed above. By deriving (19)–(20) in the next section, we provide a framework which is rather general, in that it can be applied to different quark models and flavor-mixing interactions $\mathcal{L}_I$. We illustrate its usage on the examples of the MIT bag model and instanton-induced interaction. At least in principle, this framework also puts all Lorentz channels on an equal footing, depending on which Γ is plugged into (19)–(20).

2 Nucleon strangeness induced on top of the valence quark state

It is not very surprising, in light of the nonvanishing quark scalar condensates in nonperturbative QCD, that some matrix elements $\langle N | \bar{s} \Gamma s | N \rangle$ can be markedly different from zero. The vacuum expectation value of $\bar{s}s$ is approximately as large as for non-strange quarks: $\langle 0 | \bar{s}s | 0 \rangle \approx \langle 0 | \bar{u}u | 0 \rangle = \langle 0 | \bar{d}d | 0 \rangle$, i.e., roughly equal to or even more negative than $(-200 \mathrm{MeV})^3$. The MIT bag model provides a good illustration of how this leads to a large $\langle N | \bar{s}s | N \rangle$ [5]. However, there may also be $s\bar{s}$ pairs other than those from the QCD-vacuum condensate, so that normally-ordered strange operators can, in principle, also have non-vanishing nucleon matrix elements.

Since we are interested in the $s\bar{s}$ pairs that may exist in addition to those from the (nonperturbative) vacuum channel, it is convenient to define the normal ordering with respect to the nonperturbative vacuum $|0\rangle$:

$$: \bar{q} \Gamma q := \bar{q} \Gamma q - \langle 0 | \bar{q} \Gamma q | 0 \rangle.$$
(10)

Ideally, this referent vacuum state $|0\rangle$ would be the true nonperturbative vacuum of QCD, but since in this paper we are concerned with quark models imitating QCD, (10) will mean in practice that the normal ordering is taken with respect to a model vacuum state. By this we mean the ground nucleon state from which the valence quarks are removed (for example, the "empty bag" in the case of the MIT bag model). For the strange quarks, the normal ordering with respect to this referent vacuum state $|0\rangle$ is equivalent to the normal ordering with respect to the model nucleon ground state $|N_0\rangle$ composed of the non-strange valence quarks only. Of course, such a definition of normal ordering is then necessarily tied to the characteristic hadronic scale of ~ 1 GeV, at which the nonperturbative QCD effects dominate, and at which (and also below which) quark models provide a reasonable description of the nucleon bound state.

Figure 2 illustrates how a nonvanishing value not only of $\langle N | \bar{s} \Gamma s | N \rangle$, but also of the normally-ordered $\langle N | : \bar{s} \Gamma s : | N \rangle$, can then come about: At the instant $t = t_0$ the composite nucleon is hit by an external probe (e.g., a neutrino [23]) with the coupling Γ to strange quarks. Owing to an

interaction capable of producing $s\bar{s}$ fluctuations, the nucleon state $|N\rangle$ at the time slice $t = t_0$ obviously contains not only the valence quarks uud, but also the s-quark loop to which the external probe can also couple.

Let us *schematically* write down the full nucleon (proton) state, which is also coupled to the strangeness-sensitive probe:

$$|N\rangle = \frac{1}{\mathcal{N}} \left(\sum_{X=0}^{\infty} C_X |uud\, X\rangle + \sum_{X=0}^{\infty} C_{s\bar{s}X} |uud\, s\bar{s}\, X\rangle \right)$$
$$\equiv \frac{1}{\mathcal{N}} (|N_0\rangle + |\delta N\rangle). \tag{11}$$

Here, X (starting from $X = |0\rangle$, the complicated nonperturbative QCD vacuum) symbolizes any number of various perturbative *and* nonperturbative gluon and quark configurations, including quark–antiquark pairs and, in particular, strange pairs which escape detection by this probe. These complicated configurations "dress" quarks ($q = u$, d, s, ...) into their effective counterparts, constituent quarks $\mathcal{Q} = \mathcal{U}$, $\mathcal{D}$, $\mathcal{S}$, ... In terms of the constituent quarks, this part, unperturbed by the strangeness-sensitive probe, is just the valence part $|N_0\rangle = |\mathcal{U}\mathcal{U}\mathcal{D}\rangle$ when the nucleon is a proton. It is obvious in terms of the constituent quarks that $\langle N_0| : \bar{s}\Gamma s : |N_0\rangle = 0$. The *one* strange pair detected at Γ has been explicitly denoted by $s\bar{s}$ in the $|\delta N\rangle$ part of the nucleon state perturbed by the probe. $|\delta N\rangle$ can be viewed as the response of $|N_0\rangle$ to the weakly coupled strangeness-sensitive probe. The coefficients $C_X, C_{s\bar{s}X}$ denote the amplitudes of states with various admixtures X or $s\bar{s}\, X$. $\mathcal{N}$ is the normalization. This response makes it possible for the total nucleon Γ strangeness $\langle N|\bar{s}\Gamma s|N\rangle$ to also receive a nonvanishing contribution from the nonvacuum channel $\langle N| : \bar{s}\Gamma s : |N\rangle$.

However, the question is how to get the nucleon state in sufficiently specific terms in order to have a calculable expression for $\langle N| : \bar{s}\Gamma s : |N\rangle$. A viable approach is to resort to a constituent model of hadrons. The idea of a constituent model is that the whole mess of fluctuations X is, by some model parameterization, lumped into dressing of constituent quarks $\mathcal{Q}$, as well as into effective model interactions, or a mean field they feel. The valence proton state $|N_0\rangle$ would then be identified with the model ground eigenstate $|\mathcal{U}\mathcal{U}\mathcal{D}\rangle$ built up *only* of non-strange effective quarks (so that $\langle N_0| : \bar{s}\Gamma s : |N_0\rangle = 0$, even though possibly $\langle N_0|\bar{s}\Gamma s|N_0\rangle \neq 0$, at least for $\Gamma = 1$, owing to the strange vacuum condensate). Let us denote all possible higher eigenstates of some model Hamiltonian H_0 by $|k\rangle$:

$$H_0|N_0\rangle = E_{N_0}|N_0\rangle, \quad H_0|k\rangle = E_k|k\rangle, \quad E_k > E_{N_0}. \tag{12}$$

The Hamiltonian H_0 is responsible for the formation of (model) hadron states composed of definite, fixed numbers of quarks. In the simplest case, we can imagine H_0 as consisting of a sum of one-body quark operators, typically of, say, the effective-quark kinetic-energy operator and the mean, or self-consistent, field in which the dressed valence quarks would move. For example, H_0 could be the static bag-model Hamiltonian. $|N_0\rangle$ would then be the

bag-model nucleon in its ground state, and $|k\rangle$ all higher bag states with definite numbers of constituents. In any case, H_0 defines the nucleon model – possibly together with some other ingredients (such as the confining boundary condition in bag models).

What H_0 cannot do is produce $s\bar{s}$ fluctuating pairs. To produce such pairs, we have to supplement H_0, defining the model one starts from, by some Hamiltonian H_I (corresponding to the Lagrangian density $\mathcal{L}_I$) that can produce $s\bar{s}$ excitations on top of $|N_0\rangle$. This means that $\mathcal{L}_I$, and thus also H_I, contains strange-quark field operators bilinearly, so that it can connect $|N_0\rangle$ with $|\delta N\rangle$ containing $s\bar{s}$ pairs.

To clarify that introducing $\mathcal{L}_I$ does not lead to double-counting, let us repeat that H_0 is just a model Hamiltonian, the parameters of which should mimic the effects of full, true nonperturbative QCD as much as possible. For example, if H_0 is the Hamiltonian of the nonrelativistic naive constituent quark model, it must contain the postulated mass parameter of the constituent quark mass $M_\mathcal{Q} \approx M_{N_0}/3$. The corresponding quantity in the true theory, the dynamically generated quark mass, is (in principle) the result of all possible QCD interactions, so that the interactions related to H_I can, in real QCD, also contribute to this mass by contributing to the $s\bar{s}$ fluctuations. The dynamically generated non-strange quark mass must be close to the model constituent quark mass parameter $M_\mathcal{Q}$ sitting in H_0, and only in such implicit, indirect ways are interactions like H_I present in H_0; however, they are not present explicitly, and, in fact, H_0 cannot produce any $s\bar{s}$ fluctuations at all. Therefore, if we want to study the $s\bar{s}$ fluctuations, we must introduce H_I to enrich the model nucleon with $\mathcal{S}\mathcal{S}$ fluctuations on top of $|N_0\rangle$.

In order to obtain the expression for $\langle N| : \bar{s}\Gamma s : |N\rangle$ by utilizing the Feynman–Hellmann theorem [24,25], let us define an auxiliary perturbation Hamiltonian H' by adding to H_I a source term for the strange operator we want to calculate in the "full" nucleon state $|N\rangle$:

$$H' \equiv H_I + \lambda \otimes < \bar{s}\Gamma s >, \tag{13}$$

where $< \bar{s}\Gamma s >$ is the convenient abbreviation

$$< \bar{s}\Gamma s > \equiv \int \bar{s}(x)\Gamma s(x)\, \mathrm{d}^3 x. \tag{14}$$

The generic form $\lambda \otimes \Gamma$ can mean any of the cases $\lambda \mathbf{1}_4$, $\lambda_\mu \gamma^\mu$, $\lambda_{5\mu} \gamma^\mu \gamma_5$, $\lambda_{\mu\nu} \sigma^{\mu\nu}$, etc.

Then we use the auxiliary perturbation Hamiltonian as the interaction Hamiltonian in the evolution operator $U(t_2, t_1)$. The perturbation expansion of this operator is

$$U(t_2, t_1) = 1 + \sum_{n=1}^{\infty} U^{(n)}(t_2, t_1)$$
$$= \hat{T}\left\{ 1 + \sum_{n=1}^{\infty} \frac{\mathrm{i}^n}{n!} \Big[\int_{t_1}^{t_2} :L_{\mathrm{int}}(t): \mathrm{d}t \Big]^n \right\}. \tag{15}$$

Here, $\hat{T}$ denotes the time-ordering operator and $L_{\mathrm{int}}(t) = \int \mathcal{L}_{\mathrm{int}}(\mathbf{x}, t)\mathrm{d}^3 x = -H_{\mathrm{int}}(t)$ is the interaction Lagrangian

to be replaced with the form containing the strangeness sources, as in the definition of H' (13):

$$L(t)_{\mathrm{int}} \to L'(t) = L_I(t) - \lambda \otimes < \overline{s}\Gamma s(t) >$$
$$= \int \mathrm{d}^3 x \Big[\mathcal{L}_I(x) - \lambda \otimes \overline{s}(x)\Gamma s(x) \Big].(16)$$

The Feynman–Hellmann theorem then enables one to understand the nucleon matrix elements of the strange-current densities, $\langle N | : \overline{s}\Gamma s : |N\rangle$, as the response (to the strange-current source) of the $\langle N_0(t \to +\infty)|N_0(t \to -\infty)\rangle$ transition amplitude of the model ground state $|N_0\rangle$. For example, in the case of the second-order term in (15), the substitution (16) leads to

$$U^{(2)}(+\infty, -\infty) = -\frac{1}{2}\hat{T} \int_{-\infty}^{+\infty} \mathrm{d}t \int_{-\infty}^{+\infty} \mathrm{d}t' \Big[:L_I(t)::L_I(t'):$$
$$- \lambda_\alpha :< \overline{s}\gamma^\alpha s(t) >::L_I(t'): - :L_I(t): \lambda_\beta :< \overline{s}\gamma^\beta s(t') >:$$
$$+ \lambda_\alpha \lambda_\beta :< \overline{s}\gamma^\alpha s(t) >::< \overline{s}\gamma^\beta s(t') >: \Big]. \tag{17}$$

For definiteness, the above expression for $U^{(2)}$ has been written for the *vector* strange-current density. The first-order contribution to the vector nucleon strangeness can then be obtained by considering

$$\frac{\partial}{\partial \lambda_\mu} \langle N_0 | U^{(2)}(+\infty, -\infty)|N_0\rangle \bigg|_{\lambda_\mu = 0}. \tag{18}$$

In general, for any matrix Γ in the spinor space, the strange nucleon matrix element of the full nucleon state $|N\rangle$ is, to the two lowest orders (due to the $U^{(2)}$ and $U^{(3)}$ terms), given by

$$\langle N| : \overline{s}\Gamma s : |N\rangle = \mathrm{i} \int_{-\infty}^{+\infty} \mathrm{d}t' \langle N_0|\hat{T} :< \overline{s}\Gamma s(t_0) >:$$
$$\times :L_I(t'):|N_0\rangle - \frac{1}{2} \int_{-\infty}^{+\infty} \mathrm{d}t' \int_{-\infty}^{+\infty} \mathrm{d}t''$$
$$\times \langle N_0|\hat{T} :< \overline{s}\Gamma s(t_0) >::L_I(t')::L_I(t''):|N_0\rangle. \tag{19}$$

Obviously, the nonvanishing contributions to (19) occur only when the strange-quark fields are fully contracted. For example, the integrand of the first term in (19), written in terms of space integrals over the contracted strange-current and Lagrangian densities, is

$$\int \mathrm{d}^3 x \, \mathrm{d}^3 x' \langle N_0|\hat{T} : \overline{s}(x)\Gamma s(x) :: \mathcal{L}_I(x') : |N_0\rangle$$
$$= \int \mathrm{d}^3 x \, \mathrm{d}^3 x' \langle N_0| : \overline{\overline{s}(x)\Gamma \underbrace{s(x)\mathcal{L}_I}}(x') : |N_0\rangle, \tag{20}$$

where the contractions are indicated by over- and underbraces, and $t_0 \equiv x_0$ and $t' \equiv x'_0$, for consistency of the notation. So, the first term in (19) corresponds to Fig. 2, since these contractions, or time-ordered pairings, are, of course, the propagators of strange quarks. In the second term, the two contractions must connect the
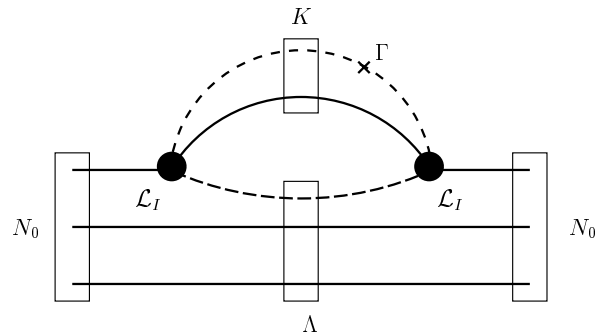

Fig. 3. A response of the valence nucleon state $|N_0\rangle$ to a strangeness source at Γ through two interactions $\mathcal{L}_I$. This type of contribution can be associated with the kaon-loop contribution to the nucleon strangeness (a possible $K\Lambda$ intermediate state is therefore indicated)

strangeness source at Γ with two different, separately-normally-ordered-interaction Lagrangian densities which act as "sinks" for strangeness at two different points of the valence–quark lines. In any case, there must be an additional strange-quark contraction between these two : $\mathcal{L}_I$:, and this completes the strange-quark loop. Fig. 3 shows an example of the graphs originating from the second term of (19), namely the $U^{(3)}$ contribution. Clearly, in this way, one can generate contributions corresponding to kaon–baryon loops in the approaches employing hadron degrees of freedom. Below, we will use a strangeness-generating interaction $\mathcal{L}_I$ that is perturbative, so that we do not expect sizable contributions to the $s\overline{s}$ effects from the second order in $\mathcal{L}_I$. In addition to that, there are indications that the contributions related to strange-meson loops should be rather small, even when one does not restrict oneself to perturbative $s\overline{s}$-generating interactions. Some of these indications come from model-dependent calculations, e.g., in the Nambu and Jona-Lasinio (NJL) model [26]. Recently, however, Geiger and Isgur presented a parameter-free analysis within a rather general framework consistent with the many empirical constraints (such as the OZI rule), which shows that a complete set of strong strange-meson–baryon loops, computed in a model consistent with the OZI rule, gives (after delicate cancellations) only small observable $s\overline{s}$ effects. We therefore do not consider the $s\overline{s}$ effects from the second order in $\mathcal{L}_I$.

3 Strangeness evaluation with a specified interaction $\mathcal{L}_I$

The evaluation of (19) is, in principle, straightforward, once one specifies two things. The first is the overall description of hadronic structure, which in practice amounts to choosing the model for the nucleon state $|N_0\rangle$; for example, choosing some mean-field Hamiltonian such as H_0 in (12). The second is the choice of the interaction (we call it $\mathcal{L}_I$) which has the role to generate $q\overline{q}$ fluctuations on top of the valence nucleon state $|N_0\rangle$ (the eigenstate of H_0). Specifying H_0 also defines the model single-quark

solutions, and we can use them as an appropriate wave-function basis to expand the quark fields $q(x)$ ($q = u, d, s$) in terms of creation ($\mathcal{U}_K^\dagger, \mathcal{D}_K^\dagger, \mathcal{S}_K^\dagger$) and annihilation ($\mathcal{U}_K, \mathcal{D}_K, \mathcal{S}_K$) operators of dressed quarks and antiquarks:

$$q(x) = \sum_K \left(\mathcal{Q}_K q_K(\mathbf{x}) e^{-i\omega_K t} + \mathcal{Q}_K^{c\dagger} q_K^c(\mathbf{x}) e^{i\omega_K t} \right) . \quad (21)$$

Here, $q_K(\mathbf{r})$ denotes a model wave function of a quark of flavour q, where K stands for the set of quantum numbers labelling a model quark state; for example, in the next section we will choose to employ the MIT bag model. Then, $q_K(\mathbf{r})$ will be the solution for the quark in the K-th mode of the MIT bag.

The field contractions in (19) lead to the sums over stationary modes of single quarks and antiquarks (such as the sums (27) and (28), evaluated in the next section), or, equivalently, to the bound-state propagators of these dressed model quarks. The sum over quark modes should naturally run up only to some typical hadron-physics low-energy cutoff $\Lambda \sim 0.6 - 1$ GeV. This cutoff on quark energies is dictated by the fact that nonperturbative interactions between quarks operate at low energies, whereas they gradually weaken and go over to the perturbative regime at higher energies. (In the aforementioned study of $s\bar{s}$ effects of kaon loops [18], Geiger and Isgur have shown the importance of high-mass intermediate states in these loops. However, since these are hadronic, meson–baryon intermediate states, this does not conflict with cutoffs such as $\Lambda \sim 1$ GeV on *quark* energies. In other words, the dominant portions of the results of [18] are accounted for by states lying below 3–3.5 GeV. For comparison, our cutoff of 1.1 GeV (see Table 1), imposed on the energies of one strange quark and one antiquark fluctuating on top of the valence nucleon state, corresponds to total energies up to $2\Lambda + M_N \sim 3$ GeV as well. This leads us to believe that we have accounted for the majority of important degrees of freedom.) The cutoff values such as ours are typical of calculations in models of low-energy QCD, e.g., the NJL model [26]. Obviously, we suppose here that the nucleon strangeness is the effect of low-energy nonperturbative QCD. Indeed, this brings us to the question of what to use concretely for $\mathcal{L}_I$ in (19) in the explicit calculation of $\langle N | : \bar{s} \Gamma s : | N \rangle$.

The Lagrangian $\mathcal{L}_I$ can, of course, be any interaction that can produce fluctuating $s\bar{s}$ pairs, but the question is which interactions can be important in producing the strangeness of the nucleon. For example, perturbative QCD interactions probed in high-energy deep inelastic scattering and revealing the sea of $q\bar{q}$ pairs, including $s\bar{s}$, should be relatively unimportant in this regard [27,28]. A theoretical analysis [29] of the CCFR data [30] on strange-quark distribution functions from neutrino–nucleon deep inelastic scattering seems to support this point of view. For example, it finds a very small upper bound on the strange radius of the nucleon ($|\langle r^2 \rangle_s| \leq 0.005$ fm^2) [29]. The possibly enhanced nucleon strangeness is thus expected (see, e.g., [27]) as an effect of nonperturbative QCD, which, at low energies, around the nucleon mass scale, is certainly more important for hadronic structure than perturbative

QCD, and can lead to $s\bar{s}$ pairs already at small momentum transfers, i.e., large distances. Nonperturbative QCD is, after all, responsible for precisely such effects as the formation of a quark–antiquark condensate $\langle 0 | \bar{q}q | 0 \rangle$ ($q = u, d, s$) and a gluon condensate that characterizes the nonperturbative QCD vacuum. Some investigators (see, e.g., [31], [3, 32], or, for comprehensive reviews, [33–35]) have suggested that instantons are among the most important nonperturbative configurations of the gluon fields. By now it has certainly been well established that the effective interaction between quarks resulting from the presence of instantons (we call this interaction $\mathcal{L}_{\mathrm{inst}}$) plays a very important role in the formation of hadron structure [33,34], although it is not responsible for confinement [36,37], as thought previously. (In the present approach, confinement must be taken care of by the unperturbed Hamiltonian H_0 anyway.) In our opinion, this $\mathcal{L}_{\mathrm{inst}}$ is therefore worth testing as an important candidate for the interactions $\mathcal{L}_I$ generating the strange nucleon matrix elements of some operators. A calculation [26] in the context of the NJL model seems to be an indication that $\mathcal{L}_{\mathrm{inst}}$ is indeed the most important part of $\mathcal{L}_I$. The calculation in [26] found that large strange-pair components were present in the nucleon only if the instanton-induced interaction was included in low-energy dynamics.

Here we quote the vacuum-averaged version of the instanton-induced interaction $\mathcal{L}_{\mathrm{inst}}$, derived by [4] in the instanton-liquid approach but transformed to the x space. It is convenient to separate it into one-, two- and three-body pieces (1) $\mathcal{L}_1, \mathcal{L}_2$ and $\mathcal{L}_3$, respectively:

$$\mathcal{L}_1 = -n \left(\frac{4\pi^2}{3} \rho^3 \right) \left\{ \mathcal{F}_u \, \bar{u}_R \, u_L + (u \leftrightarrow d) + (u \leftrightarrow s) \right\}$$
$$+ (R \leftrightarrow L) , \quad (22)$$

$$\mathcal{L}_2 = -n \left(\frac{4\pi^2}{3} \rho^3 \right)^2 \left\{ \mathcal{F}_u \mathcal{F}_d \left[(\bar{u}_R u_L)(\bar{d}_R d_L) \right. \right.$$
$$+ \frac{3}{32}(\bar{u}_R \lambda^a u_L \bar{d}_R \lambda^a d_L - \frac{3}{4}\bar{u}_R \sigma_{\mu\nu}\lambda^a u_L \bar{d}_L \sigma^{\mu\nu}\lambda^a d_L) \right]$$
$$\left. + (u \leftrightarrow s) + (d \leftrightarrow s) \right\} + (R \leftrightarrow L) , \quad (23)$$

$$\mathcal{L}_3 = -n \left(\frac{4\pi^2}{3} \rho^3 \right)^3 \mathcal{F}_u \mathcal{F}_d \mathcal{F}_s \frac{1}{3!} \frac{1}{N_c(N_c^2 - 1)}$$
$$\times \epsilon_{f_1 f_2 f_3} \epsilon_{g_1 g_2 g_3} \left\{ \frac{2N_c + 1}{2N_c + 4} (\bar{q}_R^{f_1} q_L^{g_1})(\bar{q}_R^{f_2} q_L^{g_2})(\bar{q}_R^{f_3} q_L^{g_3}) \right.$$
$$\left. + \frac{3}{8(N_c + 2)} (\bar{q}_R^{f_1} q_L^{g_1})(\bar{q}_R^{f_2} \sigma_{\mu\nu} q_L^{g_2})(\bar{q}_R^{f_3} \sigma^{\mu\nu} q_L^{g_3}) \right\}.$$
$$(24)$$

Here, n is the instanton density and $\mathcal{F}_f$ ($f = u, d, s$) are the characteristic factors (corresponding to inverse effective quark masses) composed of current light-quark masses m_f, average instanton size $\rho \simeq 1/3$ fm [38,3,32], and the

quark condensate $\langle 0|\bar{q}q|0\rangle$. For example, for the u flavour, $\mathcal{F}_u \equiv [m_u\rho - (2\pi^2/3)\rho^3\langle 0|\bar{u}u|0\rangle]^{-1}$, and analogously for the other flavours. The left (and right) projected components are defined in the usual way, $u_{L,R} = \gamma_\mp u \equiv (1/2)(1 \mp \gamma_5)u$.

In the three-body interaction $\mathcal{L}_3$, the indices f_i, g_i ($i = 1, 2, 3$) run over light flavours u, d and s. For example, $g_3 = d$ means $q_L^{g_3} = d_L$. Repeated indices are summed over. The interaction defined here by $\mathcal{L}_1, \mathcal{L}_2$ and $\mathcal{L}_3$ is actually the same as the well-known one of Shifman, Vainshtein and Zakharov (SVZ) [2], although the present three-body term (24) looks much simpler. In fact, Nowak [39] simply Fierzes away very complicated colour structures present in the SVZ interaction [2], reshuffling them to simple prefactors involving the number of quark colours N_c.

Although Nowak, et al., derives this interaction in the random instanton liquid model (RILM) with the help of the mean-field, or quenched, approximation (where the collective coordinates of instantons and anti-instantons are randomly distributed, thus neglecting potentially important correlations) and in the long-wavelength limit, their version of the interaction induced by small instantons is still considered useful even in the most recent reviews of instanton physics [35]. [4] took into account the delocalization of zero modes and long-wavelength properties (scales $> 1/3$ fm), arriving at the interaction basically corresponding to that of SVZ [2], apart from the effects of smearing over the average size ρ of a small instanton, $\rho \simeq 1/3$ fm [38,3,32]. In the limit of no smearing, the SVZ instanton-induced interaction is obtained; that is, the interaction averaged over the small instanton volume is taken to be the local interaction (22)–(24). In the long-wavelength limit, it should approximate well the intermediate-range ($\sim 1/3$ fm) QCD effects, which are already of nonperturbative origin, but are still not responsible for confinement appearing at still larger scales.

We also note that the average instanton size $\rho \simeq 1/3$ fm $= (600\text{MeV})^{-1}$ is consistent with what we have said above about the typical hadronic cutoff scale $\Lambda \sim 0.6 - 1$ GeV. Namely, the effective interaction $\mathcal{L}_{\text{inst}}$ cannot be operative at energies which would probe distances significantly smaller than the average size of these extended objects, instantons, which produce $\mathcal{L}_{\text{inst}}$.

Obviously, the two-body term is the one which, through (19) and (20), yields the graph in Fig. 2. The contribution to the nucleon strangeness due to the three-body interaction $\mathcal{L}_3$ is exemplified by the last loop in Fig. 1. Such graphs come about when contractions in (20) are performed with a strange bilinear in $\mathcal{L}_3$. The $\bar{s}s$ bilinear in the one-body term $\mathcal{L}_1$ can produce the strange-quark loops disconnected from the valence quarks.

We should also comment on the consistency of using the instanton-induced interaction $\mathcal{L}_{\text{inst}}$ for $\mathcal{L}_I$ in (19), even when we view (19) as a purely perturbative result. If we take the perturbative viewpoint, why is (19) applicable not only to parts of $\mathcal{L}_I$ which come from perturbative interactions such as the perturbative gluon exchange, but also to $\mathcal{L}_{\text{inst}}$ (22)–(24) which is of nonperturbative origin?

The point is that the *origin* of $\mathcal{L}_{\text{inst}}$ is nonperturbative, i.e., these effective interactions between quarks are the consequence of nonperturbative gluon configurations – instantons. However, $\mathcal{L}_{\text{inst}}$ itself contains a small parameter, namely the instanton density n, and it is so small that a perturbative expansion in its powers is possible. Original estimates [38], where $n \approx 1.6 \times 10^9$ MeV^4 = 1 fm^{-4}, have proved to be reliable, as they also have remained essentially unchanged [34] in the more recent instanton-liquid calculations. It is useful to define a "dimensionless instanton density" $\tilde{n}$ by expressing it in units of the average instanton size ρ, $n \equiv \tilde{n}\rho^{-4}$. The commonly accepted value is $\rho = 1/600$ MeV$^{-1} \simeq 1/3$ fm [3,32,40]. Therefore, $\tilde{n} \simeq 12.4 \times 10^{-3} \simeq 1/81$, and this dimensionless parameter indicates that the probability of finding an instanton is small. Obviously $\tilde{n}$ is small enough to be used as the parameter of the perturbative expansion. We should also keep in mind that this is the instanton density in the true, nonperturbative QCD vacuum, while in some circumstances, the appropriate n can be even smaller. Notably, [41] has found that in the MIT bag model enlarged with the instanton-induced interaction (22)–(24), which is used in the next section for the first evaluations of the nucleon strangeness using formula (19), the instanton density is very strongly depleted with respect to the true QCD vacuum.

Of course, this depletion, relative to the instanton density in the true QCD vacuum, is just the way to express the small *probability of penetration* of the instanton liquid from the RILM vacuum (modeling the true QCD vacuum outside the bag) into parts of the volume inside the otherwise perturbative bag. If one wants, this can be visualized as occasional penetration into the bag (with a small probability) by drops of the instanton liquid. It is important to note that it is not a different kind of liquid, with different properties, but that it must be the same liquid – namely the one giving rise to the interaction (22)–(24). For that reason, we use in the instanton-induced interaction $\mathcal{L}_I$ (22)–(24), as usual, the empirical value of the condensate $\langle 0|\bar{q}q|0\rangle$.

The nonvanishing (albeit small) probability for penetration of the droplets of instanton liquid from the true nonperturbative "instanton vacuum" of QCD into the "perturbative" MIT bag interior explains why, in the case of the MIT bag, we should be concerned with the strangeness coming from the one-body term $\mathcal{L}_1$. It is true that this term does not involve any interaction with valence quarks and one would thus expect that it is already included in the vacuum contribution. Nevertheless, recall that in the MIT bag model, Donoghue and Nappi [5] obtained the strangeness of the perturbative MIT bag by subtracting the (negative) nonperturbative vacuum contribution, since everything is measured with respect to the true nonperturbative QCD vacuum as the referent, "zero" level. However, if there is a nonvanishing probability for penetration of droplets of the random instanton liquid vacuum of QCD, it means that the difference of the bag interior with respect to the true nonperturbative QCD vacuum is not so large. Hence, the over-subtracted

strangeness should be put back in, and this is the reason for the one-body term $\mathcal{L}_1$ contributing to our $s\bar{s}$ pairs on top of the vacuum.

4 Instanton-induced strangeness in the MIT bag model

We now turn to the actual calculation of strange nucleon matrix elements in a specific model, with the instanton-induced interaction $\mathcal{L}_{\text{inst}}$ given by (22)–(24). For definiteness, we quote the results for the proton – the neutron case is quite similar. Using (19), we can write the proton-strangeness matrix element as

$$\langle N|:\bar{s}\Gamma s:|N\rangle = \mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}t'\,\langle N_0|\hat{T}:\int \mathrm{d}^3 x$$
$$\times\,\bar{s}(\mathbf{x},t_0)\Gamma s(\mathbf{x},t_0)::\int \mathrm{d}^3 y\,\mathcal{L}_{\text{inst}}(\mathbf{y},t'):|N_0\rangle\,,\quad (25)$$

where we have kept only the first term in the perturbation series over low instanton density. We have treated each of the three parts of $\mathcal{L}_{\text{inst}}$ (1) separately. The one-body interaction $\mathcal{L}_1$ (22) is the simplest of all. Since no valence quarks take part in this interaction, the only relevant part of $\mathcal{L}_1$ is

$$-n\left(\frac{4\pi}{3}\rho^3\right)\mathcal{F}_s[\bar{s}_R(\mathbf{y},t')\,s_L(\mathbf{y},t') + \bar{s}_L(\mathbf{y},t')\,s_R(\mathbf{y},t')]\,.\quad (26)$$

Expanding the strange-quark fields s, as in (21) and contracting them leads to the following contribution of the one-body interaction $\mathcal{L}_1$ (22) to the matrix element (25):

$$\langle N|:\bar{s}\Gamma s:|N\rangle_{\mathcal{L}_1} = 4\pi^2 n\rho^3\mathcal{F}_s\sum_{K,L}\frac{1}{\omega_K+\omega_L}$$
$$\times\left\{\int \mathrm{d}^3 x\,\bar{s}_K(\mathbf{x})\Gamma s_L^c(\mathbf{x})\int \mathrm{d}^3 y\,\bar{s}_L^c(\mathbf{y})s_K(\mathbf{y}) + (s\leftrightarrow s^c)\right\}\,.\quad (27)$$

We now choose the MIT bag as our concrete model for the nucleon[1]. Therefore, the wave functions $q_K(\mathbf{r})$ ($q = u,d,s$) denote the MIT bag model solutions[2], K stands for

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[1] The problem that the MIT bag model has with the breaking of chiral symmetry on the bag boundary is cured in various versions of the chiral bag model by containing the pion fields outside the bag with quarks, complete with appropriate boundary conditions. In the models where the bag radius is as large as in the ordinary MIT bag [42], the pion field outside is so weak that it does not significantly perturb the quark sector where our $\mathcal{L}_I$ acts, and cannot influence the strangeness much. Therefore, the results obtained in such a chirally invariant, but more complicated, model should not be very different from those obtained in the simple MIT bag model; so that in the next section, we stick to the latter for concreteness and simplicity.

[2] We follow the conventions of [43] for the MIT bag wave functions. See also our more complete account [44], where we give technicalities in detail.

Table 1. Strange-quark energy levels $\omega_{n\kappa}$, which can be excited by the instanton interaction

n	κ	$\omega_{n\kappa}$ /MeV
0	-1	514.0
0	-2	726.7
0	1	797.4
1	-1	1104.9

the set $\{n,\kappa,j_3\}$, where n is the radial excitation number, and the quantum number κ is determined by the total and orbital angular momenta j and l, respectively. ω_K is the energy of the quark in the bag state K. With all this, the one-body contribution to the nucleon strangeness is completely specified.

The sum over $K = \{n,\kappa,j_3\}$ and $L = \{n',\kappa',j_3'\}$ goes up to the state with $n = 1$, $\kappa = -1$ (corresponding to the cutoff of about 1.1 GeV), encompassing four lowest-lying strange quark states displayed in Table 1. The expression for the contribution of the two-body interaction $\mathcal{L}_2$ (23) is obtained in the same way as (27). However, it is somewhat more complicated, involving valence quark wave functions also. Luckily, the terms with $\sigma^{\mu\nu}$ cancel out, leaving us with the proton matrix element

$$\langle N|:\bar{s}\Gamma s:|N\rangle_{\mathcal{L}_2} = \frac{16}{3}\pi^4 n\rho^6\mathcal{F}_q\mathcal{F}_s\sum_{K,L,\pm}\frac{1}{\omega_K+\omega_L}$$
$$\times\left\{\int \mathrm{d}^3 x\,\bar{s}_K(\mathbf{x})\Gamma s_L^c(\mathbf{x})\int \mathrm{d}^3 y\,\bar{s}_L^c(\mathbf{y})\gamma_{\pm}s_K(\mathbf{y})\right.$$
$$\times\left[2\bar{q}_{0,-1,\frac{1}{2}}(\mathbf{y})\,\gamma_{\pm}\,q_{0,-1,\frac{1}{2}}(\mathbf{y})\right.$$
$$\left.+\,\bar{q}_{0,-1,-\frac{1}{2}}(\mathbf{y})\,\gamma_{\pm}\,q_{0,-1,-\frac{1}{2}}(\mathbf{y})\right]$$
$$+\int \mathrm{d}^3 x\,\bar{s}_K^c(\mathbf{x})\Gamma s_L(\mathbf{x})\int \mathrm{d}^3 y\,\bar{s}_L(\mathbf{y})\gamma_{\pm}s_K^c(\mathbf{y})$$
$$\times\left[2\bar{q}_{0,-1,\frac{1}{2}}(\mathbf{y})\,\gamma_{\pm}\,q_{0,-1,\frac{1}{2}}(\mathbf{y})\right.$$
$$\left.\left.+\,\bar{q}_{0,-1,-\frac{1}{2}}(\mathbf{y})\,\gamma_{\pm}\,q_{0,-1,-\frac{1}{2}}(\mathbf{y})\right]\right\}\,.\quad (28)$$

Here, $q_{0,-1,\pm\frac{1}{2}}(\mathbf{y})$ is the wave function for the ground state of the valence quark in the bag, which we take to be the same for u and d quarks.

Going now to the three-body interaction $\mathcal{L}_3$ (24), expressions become extremely long and complicated, so we do not write them down here. As seen below, it turns out that this contribution is much smaller than the preceding two anyway.

After focusing on the scalar ($\bar{s}s$) and pseudoscalar ($\bar{s}\gamma_5 s$) strangeness as the channels preferred by the QCD-

vacuum fluctuations [1], we have also checked the vector $(\bar{s}\gamma_\mu s)$ and the axial-vector $(\bar{s}\gamma_\mu\gamma_5 s)$ channels.

The calculation of the contribution of the two-body $\mathcal{L}_2$ and the three-body $\mathcal{L}_3$ instanton interactions is tedious and in the manipulation of all these formulas we have relied heavily on MATHEMATICA package [45] for symbolic computer calculations.

To illustrate how our calculations in the MIT bag model have been performed, and in which way such a model choice influences our results, we briefly sketch the calculation with the one-body part $\mathcal{L}_1$ interaction.

4.1 Scalar and pseudoscalar strangeness

Let us first consider the **scalar** strange-current density $\bar{s}s$ inside the proton. The expression for the matrix element can be written as

$$\langle N(p')|\bar{s}se^{-i\mathbf{q}\cdot\mathbf{x}}|N(p)\rangle = A_s(q^2)\bar{u}_N(p')u_N(p) , \qquad (29)$$

where $q^2 = (p-p')^2$, and u_N are nucleon spinors. $A_s(q^2)$ is the scalar form factor accounting at for the scalar strangeness of the proton at $q^2 = 0$.

Calculations inside the bag model can be performed by making the substitution $\Gamma = 1$ and inserting the appropriate quark and antiquark wave functions in (27). By a simple calculation, one can show that the surviving combination is the one with $\kappa = -1$, $\kappa' = 1$ and $\kappa = 1$, $\kappa' = -1$, and (27) reduces to

$$\langle N|:\bar{s}s:|N\rangle_{\mathcal{L}_1} = 4\pi^2 n\rho^3\mathcal{F}_s\sum_{n=0}^{1}\frac{4}{\omega_{n,-1}+\omega_{0,1}}$$

$$\times\Bigg[N_{-1}(x_{n,-1})N_1(x_{0,1})\int r^2\mathrm{d}r$$

$$\times W_+(n,-1)W_-(0,1)j_0(x_{n,-1}\frac{r}{R})j_0(x_{0,1}\frac{r}{R})$$

$$+W_-(n,-1)W_+(0,1)j_1(x_{n,-1}\frac{r}{R})j_1(x_{0,1}\frac{r}{R})\Bigg]^2 .(30)$$

The normalizations $N_{\pm1}(x_{n,\pm1})$ and the $W_\pm$ factors, related to the quark wave functions, are given in [43] and [44].

The above equation represents the contribution to the strange scalar form factor $A_s(q^2 = 0)$ coming from the one-body interaction. The remaining contributions from the $\mathcal{L}_2$ and $\mathcal{L}_3$ instanton interactions can be calculated similarly and the results are

$$\langle N|:\bar{s}s:|N\rangle_{\mathcal{L}_1} = 0.035 , \qquad (31)$$
$$\langle N|:\bar{s}s:|N\rangle_{\mathcal{L}_2} = 0.023 , \qquad (32)$$
$$\langle N|:\bar{s}s:|N\rangle_{\mathcal{L}_3} = 2.9\times10^{-4} . \qquad (33)$$

Summing them up gives

$$A_s(0)_{\mathcal{L}_{\mathrm{inst}}} = 0.058. \qquad (34)$$

The evaluation of space integrals has been performed numerically using the following values: the bag radius $R=$

$1/197.3\,\mathrm{MeV}^{-1}\approx1$ fm, the average instanton size $\rho=1/600$ MeV^{-1} and the instanton density $n = 2.66\times10^7$ MeV^4, which is the depleted instanton density in the MIT bag as found in [41]. Moreover, we have taken the strange quark mass $m_s=200$ MeV and the valence quark mass $m_u = m_d \equiv m_q=8$ MeV. The quark condensate that follows from the Gell-Mann–Oakes–Renner relation for these quark masses and the empirical meson masses is $\langle0|\bar{q}q|0\rangle\approx$ $(-200\mathrm{MeV})^3$.

The **pseudoscalar** strange form factor B_s is defined as

$$\langle N(p')|\bar{s}\gamma_5 se^{-i\mathbf{q}\cdot\mathbf{x}}|N(p)\rangle = B_s(q^2)\bar{u}_N(p')\gamma_5 u_N(p) . \quad (35)$$

For the pseudoscalar strange current $\bar{s}\gamma_5 s$, (27) gives the vanishing one-body contribution

$$\langle N|:\bar{s}\gamma_5 s:|N\rangle_{\mathcal{L}_1} = 0 . \qquad (36)$$

Analogously, we obtain the vanishing result for the other two instanton interactions, i.e., $\langle N|:\bar{s}\gamma_5 s:|N\rangle_{\mathcal{L}_{\mathrm{inst}}} = 0$. We thus obtain

$$B_s(0)_{\mathcal{L}_{\mathrm{inst}}} = 0 \qquad (37)$$

as the vanishing total instanton contribution to the pseudoscalar form factor.

4.2 Vector and axial-vector strangeness

In (5) the **vector** strangeness has been displayed in terms of the Dirac (F_1^s) and the Pauli (F_2^s) form factors. For comparison with experimental data, the (strange) Sachs form factors G_E^s (electric) and G_M^s (magnetic) are widely used:

$$G_E^s(q^2) = F_1^s(q^2) + \frac{q^2}{4M_N^2}F_2^s(q^2) ,$$
$$G_M^s(q^2) = F_1^s(q^2) + F_2^s(q^2) . \qquad (38)$$

By taking the nonrelativistic nucleon spinor

$$u_N(p,s) = \sqrt{\frac{E+M_N}{2E}}\begin{pmatrix}\chi_s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\chi_s\end{pmatrix} \qquad (39)$$

in the Breit frame defined by

$$q^\mu = (q^0,\mathbf{q}) = (0,\mathbf{q}_B) ,$$
$$\mathbf{p} = \frac{\mathbf{q}_B}{2} , \quad \mathbf{p}' = -\frac{\mathbf{q}_B}{2} , \qquad (40)$$

the components of the vector current take the form

$$\langle N(p',s')|V_0^s|N(p,s)\rangle = \frac{m}{E}\chi_{s'}^\dagger\chi_s G_E^s(-\mathbf{q}_B^2) , \qquad (41)$$

$$\langle N(p',s')|\mathbf{V}^s|N(p,s)\rangle = \frac{1}{2E}\chi_{s'}^\dagger i(\boldsymbol{\sigma}\times\mathbf{q}_B)\chi_s G_M^s(-\mathbf{q}_B^2)$$
$$. \qquad (42)$$

In order to calculate the contribution of the instanton-induced vector strange current inside the MIT bag, we

have to identify the form factors in (42) with the Fourier-transformed vector current within the bag:

$$\langle N(p')| : V_\mu^s : |N(p)\rangle_{\mathcal{L}_{\text{inst}}}$$
$$= \langle N(p')| : \int \mathrm{d}^3 r \, e^{-i\mathbf{q}_B \cdot \mathbf{r}} \bar{s}(\mathbf{r}) \gamma_\mu s(\mathbf{r}) : |N(p)\rangle_{\mathcal{L}_{\text{inst}}}, \quad (43)$$

using the static limit $q \to 0$. The check with the V_0^s component of the vector current gives zero, i.e., $G_E^s(q^2 = 0)_{\text{inst}} = 0$, which is as it should be.

A similar calculation for the space components $\mathbf{V}^s$ shows a nontrivial cancellation among the contributions of quarks in the loop with different spin orientations, producing the total result

$$G_M^s(0)_{\mathcal{L}_{\text{inst}}} = 0 \,. \quad (44)$$

This implies the vanishing strange magnetic moment

$$\mu_s = F_2^s(0) = 0 \,, \quad (45)$$

which is compatible with the recent measurements at MIT/Bates [46] and even more recent ones at TJNAF (JLab) [47].

The equation in (9) then implies that the baryomagnetic moment is

$$\mu_p^{(0)} = 1 + \kappa_p + \kappa_n = 0.88 \, \mu_N \quad (46)$$

The estimation of the **axial-vector** strangeness can be done along the same lines. The form-factor decomposition, assuming the G-parity symmetry of the strong interactions, has the form

$$\langle N(p')|\bar{s}\gamma_\mu \gamma_5 s|N(p)\rangle$$
$$= \bar{u}_N(p') \left(\gamma_\mu \gamma_5 G_1^s(q^2) + \frac{q_\mu}{2M_N} \gamma_5 G_2^s(q^2) \right) \bar{u}_N(p). \quad (47)$$

The instanton contribution to such a matrix element can be calculated as

$$\langle N(p')| : A_\mu^s : |N(p)\rangle_{\mathcal{L}_{\text{inst}}}$$
$$= \langle N(p')| : \int \mathrm{d}^3 r e^{-i\mathbf{q}_B \cdot \mathbf{r}} \bar{s}(\mathbf{r}) \gamma_\mu \gamma_5 s(\mathbf{r}) : |N(p)\rangle_{\mathcal{L}_{\text{inst}}} \quad (48)$$

and should be compared with the axial form factors defined in the Breit frame as

$$\langle N(p', s')|\mathbf{A}^s|N(p, s)\rangle = G_A^s(0) \chi_{s'}^\dagger \boldsymbol{\sigma} \chi_s \,. \quad (49)$$

Again, it turns out that the axial-vector strangeness induced by the instanton interaction is vanishing,

$$G_A^s(0)_{\mathcal{L}_{\text{inst}}} = 0 \,. \quad (50)$$

5 Discussion and conclusions

This paper deals with strange quarks at very small momentum transfers Q^2, as opposed to the high values of Q^2, where such nonvalence components of the nucleon

are undisputable, and also treatable using more standard methods of perturbative QCD and parton models. The original MIT bag model [48–50] represents a suitable starting point in predicting the low-energy properties of low-mass hadrons. In this model, R_{bag} imitates the separations $R_{\text{confining}} \sim 1$ fm at which confinement effects are important, corresponding to the confining scale $\Lambda_{\text{QCD}} \simeq 100$ to 300 MeV. Short-distance effects are supposedly taken care of by the perturbative one-gluon exchange.

However, in order to account for the effects at intermediate distances, i.e., at momentum scales $Q \sim \Lambda_{\chi\text{SB}} \simeq 0.6 - 1$ GeV, the effective interaction (1) and (22)–(24), induced by the liquid of small instantons (of the average size $\rho = 1/3$ fm) appears appropriate. Of course, the effects of the instanton-induced interactions are not included in Donoghue and Nappi's [5] naive bag-model relation (3) for the scalar nucleon strangeness, and the relative importance of this naive strangeness and the instanton effects is precisely what interests us here.

An advantage of formula (19) is that, at least in principle, it treats the scalar, pseudoscalar, vector, axial, tensor and pseudotensor nucleon strangeness in a unified manner – one just has to specify what Γ is. Within a chosen nucleon model, the evaluation of (19) would proceed in essentially the same way for each Γ, except for technical details.

In the scalar case ($\Gamma = 1$), the naive bag-model strangeness (3) is actually rather large for standard values of parameters. For our values, given at the end of Sect. 4.1, it is

$$A_s^{\text{Nbag}} \equiv -\langle 0|\bar{q}q|0\rangle V_{\text{bag}} = 4.36 \,, \quad (51)$$

which is much larger than the instanton-induced contribution (34), and dominates the summed strangeness

$$A_s \equiv A_s^{\text{Nbag}} + A_s(0)_{\mathcal{L}_{\text{inst}}} = 4.42 \,. \quad (52)$$

Owing to the use of a somewhat smaller value of the quark condensate, Donoghue and Nappi [5] obtain 3.6 for this naive strangeness, which is still rather large. A_s^{Nbag} depends very strongly on the model size parameter R_{bag} since $V_{\text{bag}} = R_{\text{bag}}^3 4\pi/3$. For example, A_s^{Nbag} would decrease by a factor of 2 if $R_{\text{bag}} = 0.8$ fm, a nucleon size which may be more acceptable, as the standard MIT bag value of 1 fm seems too large, (see, e.g., [51]). However, since the model dependence on the bag radius is similar for other presently interesting matrix elements, the model dependence largely cancels out when ratios are formed. In particular, the instanton-induced contribution (34) remains small in comparison with the naive nucleon-bag strangeness:

$$\frac{A_s^{\text{Nbag}}}{A_s(0)_{\mathcal{L}_{\text{inst}}}} \sim 75 \,, \quad (53)$$

for reasonable variations of the radius parameter.

Obviously, the contribution, due to the difference in the condensate with respect to the true, nonperturbative QCD vacuum, dominates the strangeness in the nucleon bag. Admittedly, the instanton-induced contribution of this size *would* be obtained in the calculation of (34) *if*,

inside the MIT bag, the nondepleted instanton density $n = 1.6 \times 10^9$ MeV^4 were used. However, we consider this merely a consistency check, and not an alternative description of strangeness in the MIT bag. This is because using the instanton density appropriate to the nonperturbative QCD vacuum containing the large-quark condensate would imply that the nonperturbative QCD vacuum and the quark condensate are assumed to be not only outside, but also inside the bag. This would indeed enable $A_s(0)_{\mathcal{L}_{\text{inst}}}$ to replace A_s^{Nbag} in full, but would also make the MIT bag description inconsistent [41].

The scalar strangeness is special because of nonvanishing scalar $\bar{q}q$ condensates of the QCD vacuum; this makes it more natural that scalar strangeness is larger than vector, axial, or other kinds of strangeness. This is especially clear in our approach applied to the MIT bag model. In this model, the scalar strangeness comes mostly from the change of the scalar $\bar{q}q$ condensates in the true QCD vacuum and their absence in the perturbative vacuum inside the cavity [5], while only the relatively small remainder in the present paper comes from the response of the valence ground state to the strangeness-sensitive probe. However, such a response is all that exists in the case of the pseudoscalar, vector, axial, etc., nucleon strangeness, since there are no pseudoscalar, vector, axial, etc., QCD-vacuum condensates either inside or outside the cavity. Since such responses tend to be much smaller than the nonperturbative vacuum contributions, significant differences in magnitude between the scalar and other kinds of strangeness are very natural in our approach. In fact, in the present case of the MIT bag model, we find the vanishing first-order contribution to the vector strangeness. The vanishing first-order contributions are also found for the pseudoscalar and axial strangeness of the nucleon.

Thus, our results confirm the conjecture of [1] for the case of the scalar strangeness.

Thus it is understandable why the results for the "non-scalar" strange quantities, such as the strangeness nucleon magnetic form factor [10–14,16,17, 19–22] or the strangeness electric mean-square radius [10, 13–22] vary so much, even by the sign, from one model to another: The non-scalar strange quantities should all be rather small, and artifacts of various models very easily put it on either side of the zero.

Our results are also consistent with the most recent measurements of the strange vector form factors at low momentum transfer, $Q^2 \lesssim 1$ GeV. The experimental strange magnetic form factor of the nucleon at $Q^2 = 0.1$ (GeV/c)^2, $G_M^s = 0.23 \pm 0.37 \pm 0.15 \pm 0.19 \, \mu_N$, obtained at MIT/Bates [46] is consistent with the absence of strange quarks, but the error bars are large. However, the main conclusion of our approach, that channels other than the scalar one should not be appreciably affected by strange quarks, seems to get support especially from the most recent and very precise TJNAF (JLab) measurement [47], which yields the small strange-vector form factors of $Q^2 = 0.48$ (GeV/c)^2, $G_E^s + 0.39 G_M^s = 0.023 \pm 0.034 \pm 0.022 \pm 0.026$ nm Furthermore, the HAPPEX Collaboration [47] plans to improve the accuracy of this result by a

factor of two in 1999. Nevertheless, its small central value, which is consistent with zero, and small errors already exclude some of the more generous predictions [10,21] for the strangeness (but not [52,14] for example).

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